Introduction to Logic

PHIL 240
Sections 501 – 509
08 September 2014

Validity and form

As we noted last week, deductive validity is simply a function of the form of the argument. In order to more easily study argument form, and thus recognize similarity among arguments, logicians employ formal languages. This week we will be constructing a formal language for sentential logic (also known as propositional logic).

A formal language for Sentential Logic

- **Vocabulary**: The “building blocks” of the language consist of the following:
  - **Sentence Letters**
  - **Connectives**
  - **Parentheses**

Sentence Letters

- We will use capital letters to represent simple sentences. So the following are "sentence letters:"

  A, B, C, … Z

  (If we need more than 26 options, we can add subscripts to give us an infinite number of options, e.g. A_0, … Z_0, A_1, … Z_1, ….)

Sentence Letters cont.

- Sentence letters are sometimes also known as **sentence variables**, because we use them to stand for sentences of natural languages.

Connectives

- Sentential connectives, called "connectives," are used to represent words in natural language that serve to connect declarative sentences. They include the following:

~, &, v, →, ↔

Tilde

- The **tilde** corresponds to the English "It is not the case that ...". (It may seem odd to term it a "connective," as it is not connecting two declarative sentences, but we will continue to use the term "connective" to apply to all of the logical operators in our system for the sake of simplicity.

~

Ampersand

- The **ampersand** corresponds to the English "Both … and …"

&

Wedge

- The **wedge** corresponds to the English "Either … or …" in its *inclusive* sense.

v

Arrow

- The **arrow** corresponds to the English "If …, then …"

$$\rightarrow$$

Double-Arrow

- The **double-arrow** corresponds to the English "if and only if."

$$\leftrightarrow$$

Parentheses

- We use the right and left parentheses as punctuation marks for the language.

) and (

Expression

- An **expression** of sentential logic is any sequence of sentence letters, sentential connectives, or left and right parentheses.

  Examples:
  (P&Q)vR is an expression
)QvRS is an expression
  (3 & 9) is NOT an expression

Metavariables

- Our text uses the metavariables Φ (Phi) and Ψ (Psi) to represent *expressions* in the language. These metavariables are not components of the language, but merely devices used to aid in explaining the language in what is, hopefully, a less confusing manner.

Well Formed Formula

- Our language for sentential logic recognizes six different types of sentences. These sentences must accord with the following seven rules in order to be considered as a **W**ell **F**ormed **F**ormula (WFF, pronounced "woof").

Atomic Sentence

1. Any sentence letter standing alone is a WFF, and is known as an Atomic Sentence, or Atom.

   The Following are Atoms:

   P
   Q

Negation

2. If Φ is an expression in the language, then the expression ~ Φ is also a WFF, and known as an negation.

   (The basic idea is that you take any sentence which is well-formed, prefix it with a tilde, and you create a new type of sentence. This sentence is known as a negation.)

   Examples: ~A
   ~P

Conjunction

3. If Φ is a WFF, and Ψ is a WFF, then the expression (Φ & Ψ) is a WFF, and the sentence is known as a conjunction.

   (This is the first of our four binary connectives. All WFFs created through the use of binary connectives require external parentheses in order to be well formed.)

   Examples: (A & B)
   (P & S)

Disjunction

4. If Φ is a WFF, and Ψ is a WFF, then the expression (Φ v Ψ) is a WFF, and the sentence is known as a disjunction.

   Examples:

   (A v B)
   ((A & B) v C)
   (A v (B v C))

Conditional

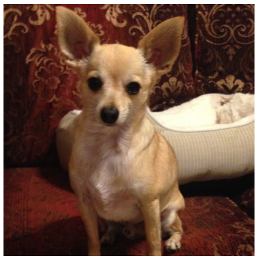
5. If Φ is a WFF, and Ψ is a WFF, then the expression ($\Phi \rightarrow \Psi$) is a WFF, and the sentence is known as a conditional. The expression to left of the $\rightarrow$ is known as the **antecedent**, and that to the right is known as the **consequent**.

Biconditional

6. If Φ is a WFF, and Ψ is a WFF, then the expression ($\Phi \leftrightarrow \Psi$) is a WFF, and the sentence is known as a biconditional.

Nothing else

7. Nothing else is a WFF, not even this little guy (who happens to say "Woof" a lot):



Parentheses Dropping Convention

Our text makes liberal use of a convention whereby parentheses may be dropped for the sake of convenience. A couple of things to note:

- You can only drop parentheses to the point that it is still clear what is the main connective operating over the sentence, according to the convention.
- Sentences employing the convention are NOT WFFs, merely abbreviations for WFFs.

Parentheses Dropping Convention (cont.)

The convention develops a hierarchy of strengths, from strongest to weakest, that may be employed to add parentheses back in order to rebuild a WFF from the abbreviation. This hierarchy is:

~

& and **v**

→

↔

Parentheses Dropping Convention (cont.)

- To generate a WFF from an abbreviation, using the convention, you:

1. Identify all connectives, noting which binary connectives are without parentheses
2. Add a pair of parentheses around each set of statements joined by a binary connective, working from strongest to weakest.