

Name (5 points): _____ Section (5 points): _____

Section I True / False questions (2 points each)

1. TRUE Any argument that is sound is also valid.
2. FALSE If the premises of an argument are all true, then that argument is sound.
3. TRUE Every universal wff contains at least one variable.
4. TRUE Every negation is a denial.
5. FALSE If the conclusion of a valid argument is false, then its premises must all be false.
6. FALSE Every WFF contains at least one connective..
7. FALSE Every WFF of predicate logic contains at least one quantifier.
8. FALSE Some conditionals are negations.
9. FALSE Every argument has at least two premises.
10. FALSE Every WFF that starts with the symbol $\forall x$ is an existential.

Section II Mark the correct completion (5 points each)

1. The conclusion of a valid argument ...
 - (a) _____ must be false if the premises are all false.
 - (b) X must be true if the premises are all true.
 - (c) _____ must be false if all the premises are true.
 - (d) _____ must be true if all the premises are false.
 - (e) _____ must be true if it is also false.
2. The following is NOT a condition on the application of $\exists E$...
 - (a) X the instantial name must occur in the line which is repeated.
 - (b) _____ the instantial name cannot occur in the line containing the sentence which is repeated.
 - (c) _____ the instantial name cannot occur in the line that motivates the assumption to be discharged.
 - (d) _____ the instantial name cannot occur in the assumption set of the line containing the sentence which is repeated save for the assumption itself.

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3. A finite interpretation may contain all but ...

- (a) _____ a universe
- (b) _____ predicate extensions
- (c) _____ truth value specifications
- (d) X a galaxy

Section III Translations (5 points each)

Using the following translation scheme, construct a strictly correct translations that includes all parentheses.

$Bx =$ 'x is a book'

$Hx =$ 'x is a hardback'

$Px =$ 'x is a paperback'

$P =$ Peter Dances

$Q =$ Jane Dances

1) Peter dances if and only if Jane Dances.

$$(P \leftrightarrow Q)$$

2) All books are paperbacks.

$$\forall x(Bx \rightarrow Px)$$

3) All books are either paperback or hardback.

$$\forall x(Bx \rightarrow (Px \vee Hx))$$

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Section IV Proofs (5 points each)

Give a proof for each of the following sequents. You may use both primitive and derived rules.

1. $\forall x(Fx \rightarrow Gx), \forall x(Gx \rightarrow Hx), \forall xFx \vdash \exists xHx$

1	(1)	$\forall x(Fx \rightarrow Gx)$	A
2	(2)	$\forall x(Gx \rightarrow Hx)$	A
3	(3)	$\forall xFx$	A
1	(4)	$(Fa \rightarrow Ga)$	1 $\forall E$
3	(5)	Fa	3 $\forall E$
1,3	(6)	Ga	4,5 $\rightarrow E$
2	(7)	$(Ga \rightarrow Ha)$	2 $\forall E$
1,2,3	(8)	Ha	6,7 $\rightarrow E$
1,2,3	(9)	$\exists xHx$	8 $\exists I$

2. $(P \rightarrow R), (\sim R \vee Q), (Q \rightarrow S) \vdash (P \rightarrow S)$

1	(1)	$(P \rightarrow R)$	A
2	(2)	$(\sim R \vee Q)$	A
3	(3)	$(Q \rightarrow S)$	A
4	(4)	P	A (for $\rightarrow I$)
1,4	(5)	R	1,4 $\rightarrow E$
1,2,4	(6)	Q	2,5 $\vee E$
1,2,3,4	(7)	S	3,6 $\rightarrow E$
1,2,3	(8)	$(P \rightarrow S)$	7 $\rightarrow I$ (4)

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3. $\forall x(Dx \rightarrow Ax), \forall x(Ax \rightarrow Cx), \exists xDx \vdash \exists xCx$

1	(1)	$\forall x(Dx \rightarrow Ax)$	A
2	(2)	$\forall x(Ax \rightarrow Cx)$	A
3	(3)	$\exists xDx$	A
4	(4)	Da	A (for $\exists E$ on 3)
1	(5)	$(Da \rightarrow Aa)$	1 $\forall E$
1,4	(6)	Aa	4,5 $\rightarrow E$
2	(7)	$(Aa \rightarrow Ca)$	2 $\forall E$
1,2,4	(8)	Ca	6,7 $\rightarrow E$
1,2,4	(9)	$\exists xCx$	8 $\exists I$
1,2,3	(10)	$\exists xCx$	3,9 $\exists E$ (4)

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Section V Truth Tables (5 points each)

Using the truth table method, with either a full truth table or an indirect truth table, determine whether or not the following sequent is valid. If it is valid, state that it is valid. If it is invalid, provide an invalidating assignment.

$$\sim(\sim P \vee Q) \vdash (P \leftrightarrow \sim Q)$$

P	Q	~	(~	P	v	Q)	⊢	(P	↔	~	Q)
T	T	F		F		T						F	F		
T	F	T		F		F						T	T		
F	T	F		T		T						T	F		
F	F	F		T		T						F	T		

Valid

Section VI Finite Interpretations (2 points each)

For each of the sentences below, indicate whether it is true or false in this finite interpretation:

U: {a, b, c}

F: {a}

G: {a,b, c}

L: {<a,a>, <a,b>, <b,c>, <c,a>}

1. FALSE $\exists x \forall y Lxy$
2. TRUE $\exists x (Fx \ \& \ Gx)$
3. FALSE $(\exists x Gx \rightarrow \forall x Fx)$
4. TRUE $\exists x Lxx$
5. TRUE $\forall x Gx$

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Section VII Probability (2 points each)

For each of the questions below, assume fair coin (Heads/Tails), fair dice (six sides numbered 1-6), and fair standard deck of cards.

1. What is the probability of getting heads on three successive flips of a coin?

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$$

2. What is the probability of not getting a 4 on a single roll of a 6-sided die?

$$1 - \frac{1}{6} = \frac{5}{6}$$

3. What is the probability of drawing, on a single draw from a standard deck of cards, either a queen or a heart?

$$\left(\frac{4}{52} + \frac{13}{52}\right) - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

4. What is the probability of drawing, on a single draw from a standard deck of cards, either a jack or a king?

$$\frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \frac{2}{13}$$

5. What is the probability of drawing two spades, on two successive draws from a standard deck of cards, if no cards are replaced?

$$\frac{13}{52} \times \frac{12}{51} = \frac{156}{2652} = \frac{13}{221}$$